Significant infrastructure changes are currently being implemented in power system networks around the world by maximizing the penetration of renewable energy, installing new transmission lines, adding flexible loads, and promoting independence in power production by disintegrating the grid into microgrids [1]. The shift of energy supply from large central generating stations to smaller producers, such as wind farms, solar photovoltaic (PV) farms, rooftop PV systems, and energy-storage systems collectively known as distributed energy resources (DERs) or inverter-based resources, is accelerating at a rapid pace. Hundreds of power electronic devices are being added, creating hundreds of new control points in the grid.

Summary

This article presents an end-to-end differential algebraic model of a power system in its entirety, including synchronous generators, wind farms, solar farms, energy storage, power electronics converters, and controllers for each device. Distributed energy resources (DERs) and power electronics devices are shown to affect small-signal stability and the dynamic performance of the grid. The article then presents a two-layer control architecture for future grids, where one layer consists of decentralized plug-and-play controllers for power electronics converter control of DERs, and another layer is composed of system-wide distributed controllers actuated through the generators.
These additions are being complemented by equal progress in sensing technology, whereby high-precision, high-sampling-rate, GPS-synchronized dynamic measurements of voltages and currents are now available from sensors like phasor measurement units (PMUs) [2]. With all of these transformational changes in the grid, operators are inclined to explore new control methods that go far beyond how the grid is currently managed (see “Summary”). A list of these revolutionary developments is presented in Table 1.

Current state-of-the-art power system controllers (especially those responsible for transient stability and power oscillation damping) are all operated in a decentralized and uncoordinated fashion using only local output feedback. A survey of these controllers is provided in “Brief Survey on Control of Synchronous Generators.” With rapid modernization of the grid, however, these local controllers will not be tenable over the long term. Instead, system-wide coordinated controllers will become essential. Such controllers, where signals measured at one part of the grid are communicated to remote parts for feedback, are called wide-area controllers [3].

Wide-area control (WAC) alone, however, will not be enough either. (See Table 2 for a list of various other defined acronyms.) It may improve the stability and dynamic performance of the legacy system, but it will not be able to keep pace with the unpredictable rate at which DERs are being added to the grid. Every time a new DER is added, it will be almost impossible for an operator to retune all of the WAC gains to accommodate the change in dynamics.

DERs have high variability and intermittency and must be operated in a plug-and-play fashion. Accordingly, their controllers must be local, decentralized, and modular in both design and implementation.
The article's second objective is to present a comprehensive list of mathematical models of the various components of a power grid, ranging from synchronous generators, their internal controllers, loads, wind and solar farms, batteries to the power electronic device interfaces, and associated control mechanisms for each of these components. While these models are individually well cited in the literature, few references have collected all of them together to understand the holistic dynamic behavior of an entire grid.

The rest of the article is organized as follows. The next section describes the dynamic model of a power system integrated with different types of DERs. A general framework for modeling is provided first, followed by details of each individual component model. The “Impact of Distributed Energy Resources on Power System Dynamics and Stability”
section demonstrates DER influence on power system dynamics through numerical simulations. Motivated by these simulation results, the “New Approaches for Controlling Power Systems With Distributed Energy Resources” section shows newly developed decentralized DER control laws using the idea of retrofit control [4] as well as distributed wide-area controllers for the damping of low-frequency oscillations using sparse optimal control [5]. The effectiveness of this combined control strategy is exhibited using an IEEE 68-bus power system with wind and solar farms. The article concludes with a list of open research problems.

**POWER SYSTEM MODELS**

First, the dynamic models of the four core components of a power system are developed—generation, transmission, load, and energy storage. The generating units are classified into conventional power plants and DERs, such as wind and PV generators. Each model follows from first principles of physics. Note that, in reality, a generation facility (whether conventional or wind/solar generation) and energy-storage entities themselves contain many generating units and storage devices. In the following, the terms generator, wind farm, solar farm, and energy-storage system refer to an aggregate of...
**Brief Survey of Wide-Area Control**

Recent research has proposed the design of wide-area control (WAC) for power oscillation damping based on optimal control [S22], linear matrix inequalities and conic programming [S24], model predictive control [S25], model reduction and control inversion [S26], and adaptive control [S27]. A tutorial on these different methods was recently reported in [3]. In [S28], WAC that can be resilient to failures was proposed. One drawback of these methods is that they usually result in a centralized implementation that can be computationally challenging. Centralized control is also not very resilient to cyberattacks [S28], [S29]. Thus, in recent years, power system operators are inclining more toward distributed WAC, where the communication graph among controllers is sparse [S30], [S31]. The work in [S30] uses geometric measures for the selection of control loops, whereas [S31] uses a sparsity-promoting linear-quadratic-regulator (LQR)-based optimal control strategy. LQR is often chosen as the central design tool, as it provides flexibility in damping selected ranges of frequencies. A real-time version of the sparse LQR design was proposed in [5] using spectral decomposition of online phase-space measurement. An advantage of this method over those in [S30] and [S31] is that it can provide sparser wide-area controllers than those techniques with a comparable closed-loop performance. See [5] for a more detailed discussion, based on numerical simulations.

**REFERENCES**


**TABLE 3** The tie-line parameters for constructing $Y$ of the IEEE 68-bus power model, which is a benchmark model used in the simulation, are available in [7]. All variables are considered to be per unit unless otherwise stated.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Numerical Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$120\pi$</td>
<td>Base angular speed (rad/s) for a 60-Hz power system</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of buses</td>
<td></td>
</tr>
<tr>
<td>$V_k \in \mathbb{C}$</td>
<td>$k$th bus voltage</td>
<td></td>
</tr>
<tr>
<td>$P_k, Q_k$</td>
<td>Active and reactive power injected from the $k$th component</td>
<td></td>
</tr>
<tr>
<td>$x_k$</td>
<td>State of the $k$th component</td>
<td></td>
</tr>
<tr>
<td>$u_k$</td>
<td>Control input of the $k$th component</td>
<td></td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>Model parameter depending on operating point</td>
<td></td>
</tr>
<tr>
<td>$Y \in \mathbb{C}^{N \times N}$</td>
<td>Admittance matrix</td>
<td></td>
</tr>
<tr>
<td>$N_G, N_L, N_W$</td>
<td>Index set of the buses connecting to generators, loads, wind farms, solar farms, energy storage, and that of nonunit buses; these sets are disjoint, and $N_G \cup N_L \cup N_W \cup N_S \cup N_E \cup N_N = {1, \ldots, N}$</td>
<td></td>
</tr>
</tbody>
</table>

For $k \in \{1, \ldots, N\}$, the nomenclature of this model is summarized in Table 3. Details of the two functions $f_k(\cdot, \cdot, \cdot)$ and $g_k(\cdot, \cdot)$ for each component will be described shortly. Throughout the article, complex variables will be written in bold fonts (for example, $\mathbf{V}$). All symbols with superscript $*$ will denote setpoints.

those individual units representing the overall facility. Similarly, the term load refers to an aggregate of all consumers inside the associated demand area.

Each aggregated unit includes its own individual bus, such as generator or load buses. The buses are connected through a network of power transmission lines. The power system may also contain buses where no generator, wind/solar farm, load, or energy-storage system is connected. These are called nonunit buses. The term component refers to either a unit with its bus or the nonunit bus. An example of these connected components is depicted in Figure 1.

As will be shown in the following, a general form for the dynamic model of the $k$th component of a power system (whether that component be a generator, load, storage, wind farm, or solar farm) can be written as

$$\Sigma_k: \begin{cases} \dot{x}_k = f_k(x_k, V_k, u_k; \alpha_k), \\ P_k + Q_k = g_k(x_k, V_k; \alpha_k) \end{cases} \quad (1)$$

for $k \in \{1, \ldots, N\}$. The nomenclature of this model is summarized in Table 3. Details of the two functions $f_k(\cdot, \cdot, \cdot)$ and $g_k(\cdot, \cdot)$ for each component will be described shortly. Throughout the article, complex variables will be written in bold fonts (for example, $\mathbf{V}$). All symbols with superscript $*$ will denote setpoints.
The $N$ components are interconnected by a transmission network. Let $Y \in \mathbb{C}^{N \times N}$ denote the admittance matrix of the network (for details of the construction of this matrix, see “Construction of Admittance Matrices”). The power balance across the transmission lines follows from Kirchhoff’s laws as

$$0 = (YV_{1:N})^* V_{1:N} - (P_{1:N} + jQ_{1:N}),$$

where $*$ is the element-wise multiplication, $^*$ is the element-wise complex conjugate operator, and $V_{1:N}$, $P_{1:N}$, and $Q_{1:N}$ are the stacked representations of $V_k$, $P_k$, and $Q_k$ for $k \in \{1, \ldots, N\}$. From (2), $V_{1:N}$ is determined for a given $P_{1:N}$ and $Q_{1:N}$. The overall dynamics of a power system can be described by the combination of (1) and (2). A signal-flow diagram of this model is shown in Figure 2.

The power system model described by (1) and (2) is operated around its equilibrium, which is determined as follows. The steady-state value of $x_k$, $V_k$, $P_k$, and $Q_k$ and parameter $a_k$ in (1) must satisfy (2) and

$$\begin{align*}
0 &= f_k(x_k, V_k, 0; a_k), \\
P_k + jQ_k &= g_k(x_k, V_k; a_k). 
\end{align*}$$

The steady-state value of $u_k$ is assumed to be zero without loss of generality. A standard procedure for finding the steady-state values consists of two steps: power flow calculation and initialization, which are summarized as follows.

- **Power flow calculation**: Find $V_{1:N}$, $P_{1:N}$, and $Q_{1:N}$ satisfying (2) and other constraints for individual components (see “Brief Tutorial on Power Flow Calculation” for the details of these constraints).
- **Initialization**: For $k \in \{1, \ldots, N\}$, given $V_k$, $P_k$, and $Q_k$, find $x_k$ and $a_k$ satisfying (3). These solutions then serve as the initial conditions for the dynamic model described by (1) and (2) for any incoming contingency.
Note that there exist an infinite number of solutions satisfying (2). However, $V_i, P_i$ and $Q_i$ of some of the components are either known a priori or specified by economic dispatch [6]. The details of this are described in “Brief Tutorial on Power Flow Calculation.” Once the steady-state values of $V_{1:N}, P_{1:N}$, and $Q_{1:N}$ are obtained, the setpoints $x_i$ and $a_k$ of the $k$th component can be computed independently in the second step. The details of this initialization step will be described later in each section describing the detailed dynamics of the components. The uniqueness of the solution $(x_i, a_k)$ satisfying (3) under a given triple $(V_i, P_i, Q_i)$ depends on the component itself. In fact, the equilibria for the generators, loads, and nonunit buses are unique. However, those for wind farms, solar farms, and energy-storage systems are not. The details of this uniqueness property will also be described in the following sections.

Next, the state-space models of the generators, loads, energy-storage systems, wind farms, solar farms, and nonunit buses conforming to the structure in (i) are derived. For easier understanding, each section starts with a qualitative description of the respective component model, followed by its state-space representation. Some parts may refer to equations that appear later in the text. To simplify the notation, the subscript $k$ is omitted unless otherwise stated.

**Generators**

A generator consists of a synchronous machine, an energy supply system (or prime mover), and an excitation system [6]. The excitation system imposes currents in the excitation winding, thereby magnetizing the rotor. The prime mover generates mechanical power to rotate the rotor in this magnetic field. The synchronous machine converts the mechanical power to electrical power, which is transmitted to the rest of the grid. The dynamics of the prime mover are usually ignored because of its slow time constant [8]–[10].

**SYNCHRONOUS MACHINE**

While various types of synchronous machine models are available in the literature [6], the well-known one-axis model or flux-decay model is used in this article; it consists of the electromechanical swing dynamics (4) and electromagnetic voltage dynamics (5). For simplicity, the mechanical power $P_m$ in (4) is assumed to be constant.

**EXCITATION SYSTEM**

The excitation system typically consists of an exciter, an automatic voltage regulator (AVR) that regulates the generator voltage magnitude to its setpoint value, and a power system stabilizer (PSS) that ensures the power system stability. The exciter (with an AVR) is modeled as (6), where $u$ is a control input representing an additional voltage reference signal to the AVR. The PSS is a typical speed-feedback-type controller that consists of two-stage lead–lag compensators and one high-pass washout filter [11].

The state-space representation of the overall generator model can be written as follows (definitions of $\dot{\omega}, \dot{V}, P + jQ$, and $u$ are given in Table 3, while those of the other symbols are provided in Table 4).

**Synchronous machine:**

Electromechanical swing dynamics

\[
\begin{align*}
\dot{\delta} &= \dot{\omega} \Delta \omega, \\
\Delta \dot{\omega} &= \frac{1}{M} 
\left( P_m - d \Delta \omega - \frac{|V|^2}{X_d} \sin(\delta - \angle V) 
+ \frac{[|V|^2]}{2} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) \sin(2\delta - 2\angle V) \right).
\end{align*}
\]

**TABLE 4 The nomenclature for the generator model.**

The values of the synchronous machine parameters below are typical and are rated at generator capacity. The parameters for the IEEE 68-bus test system (which is a benchmark power system model used later in this article) are available in [7]. AVR: automatic voltage regulator; PSS: power system stabilizer.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Numerical</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Rotor angle (rad) relative to the frame rotating at $\dot{\omega}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \omega$</td>
<td>Frequency deviation, that is, rotor angular velocity relative to $\dot{\omega}$</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>q-axis voltage behind transient reactance $X_d$</td>
<td></td>
</tr>
<tr>
<td>$V_{is}$</td>
<td>Field voltage</td>
<td></td>
</tr>
<tr>
<td>$\zeta \in \mathbb{R}^3$</td>
<td>PSS state</td>
<td></td>
</tr>
<tr>
<td>$\xi_{af}$</td>
<td>Exciter field voltage</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>PSS output</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>Additional voltage reference to AVR</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>30</td>
<td>Inertia constant (seconds)</td>
</tr>
<tr>
<td>$d'$</td>
<td>0.1</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>$\tau_{do}$</td>
<td>0.1</td>
<td>d-axis transient open-circuit time constant (seconds)</td>
</tr>
<tr>
<td>$X_a, X_q$</td>
<td>1.8</td>
<td>d- and q-axis synchronous reactance</td>
</tr>
<tr>
<td>$X_d$</td>
<td>0.3</td>
<td>d-axis transient reactance</td>
</tr>
<tr>
<td>$P_m$</td>
<td>8.0</td>
<td>Steady-state mechanical power</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>0.05</td>
<td>Time constant of exciter (seconds)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>20</td>
<td>AVR gain</td>
</tr>
<tr>
<td>$K_{pss}$</td>
<td>150</td>
<td>PSS gain</td>
</tr>
<tr>
<td>$V_{is}$</td>
<td>1.0</td>
<td>Setpoint for the field voltage</td>
</tr>
<tr>
<td>$</td>
<td>V</td>
<td>$</td>
</tr>
<tr>
<td>$\tau_{pss}$</td>
<td>10</td>
<td>Washout filter time constant (seconds)</td>
</tr>
<tr>
<td>$\tau_{L1}, \tau_{L2}$</td>
<td>0.02, 0.07</td>
<td>Lead-lag time constants of the first stage of PSS (seconds)</td>
</tr>
<tr>
<td>$\tau_{L3}, \tau_{L4}$</td>
<td>0.02, 0.07</td>
<td>Lead-lag time constants of the second stage of PSS (seconds)</td>
</tr>
</tbody>
</table>
Construction of Admittance Matrices

The following provides a brief tutorial for constructing the admittance matrix \( Y \) from tie-line data typically shown in the literature (for example, [7]). For this purpose, a power network consisting of three buses [as shown in Figure S1(a)] is considered, where \( V_i \) is the \( i \)th bus voltage and \( I_i \) is the current injected from the bus. Typically, each branch is modeled as a so-called \( \Pi \) type circuit, as shown in Figure S1(b), where \( x_i \) is the branch impedance and \( y_i \) is the shunt admittance representing a capacitance between the transmission line and ground. The values of \( x_i \) and \( y_i \) for all branches are given data.

The bus voltage and current must satisfy Kirchhoff’s law, that is,

\[
I_{1:3} = \mathbf{Y} \mathbf{V}_{1:3}, \quad I_{1:3} := [I_1, I_2, I_3]^T, \quad \mathbf{V}_{1:3} := [V_1, V_2, V_3]^T. \tag{S1}
\]

```latex
\begin{align*}
\mathbf{E} &= \frac{1}{\tau_e} \left( \frac{X_d}{X_q} E + \left( \frac{X_d}{X_q} - 1 \right) \mathbf{V} \cos(\delta - \angle \mathbf{V}) + V_{id}\right), \\
\mathbf{P} + j \mathbf{Q} &= \frac{\mathbf{E}}{X_d} \left( \frac{\mathbf{V}}{X_d} - \frac{1}{X_d} \frac{\mathbf{V}}{X_q} \sin(2\delta - 2\angle \mathbf{V}) \right) \\
&\quad + j \left( \frac{\mathbf{E}}{X_d} \cos(\delta - \angle \mathbf{V}) \right) \\
&\quad - \left| \mathbf{V} \right|^2 \left( \frac{\sin^2(\delta - \angle \mathbf{V})}{X_q} + \frac{\cos^2(\delta - \angle \mathbf{V})}{X_d} \right). \tag{5}
\end{align*}
```

Excitation system

- Exciter with AVR

\[
\begin{align*}
\frac{1}{\tau_e} &= \frac{1}{\tau_e} \left( -V_{id} + V_{id} + V_{el}\right), \\
V_{el} &= K_e \left( |\mathbf{V}| - |\mathbf{V}| - v + n_0\right), \tag{6}
\end{align*}
\]

where \( |\mathbf{V}| \) represents the setpoint of \( |\mathbf{V}| \).

- PSS

\[
\dot{\zeta} = A_{pss} \zeta + B_{pss} \Delta \omega, \quad v = C_{pss} \zeta + D_{pss} \Delta \omega, \tag{7}
\]

Note that (S1) is equivalent to (2) when \( N = 3 \) by taking the conjugate of (S1) and multiplying each \( k \)th row of (S1) by \( \mathbf{V}_k \). Thus, \( \mathbf{Y} \) in (S1) is the matrix that must be constructed. The calculation of its (1,1) element, denoted by \( Y_{11} \), is as follows. Equation (S1) should hold when \( V_2 = V_3 = 0 \). The first line in (S1) then becomes \( I_1 = \mathbf{Y}_{11} \mathbf{V}_1 \). Alternatively, when \( V_2 = V_3 = 0 \), it follows from Figure S1(a) that \( I_1 = (y_1 + x_1^*) \mathbf{V}_1 \). Thus, \( Y_{11} = y_1 + x_1^* \).

Repeating this for all of the elements, the admittance matrix is constructed as

\[
Y = \begin{bmatrix}
y_1 + x_1^* & 0 & -x_1^* \\
0 & y_2 + x_2^* & -x_2^* \\
-x_1^* & -x_2^* & y_1 + y_2 + x_1^* + x_2^*
\end{bmatrix}. \tag{S2}
\]

**FIGURE S1** (a) A simple power system consisting of three buses. (b) The \( k \)th branch model as a \( \Pi \)-type circuit.
\[ x_k = [\delta_k, \Delta \omega_k, E_k, V_{d,k}, V_{q,k}]^T \in \mathbb{R}^7, \]
\[ \alpha_k = [P_{m,k}, V_{d,k}, V_{q,k}]^T \in \mathbb{R}^3, \]
and \( f_i(\cdot, \cdot, \cdot) \) and \( g_i(\cdot, \cdot, \cdot) \) in (1) follow from (4) to (8). The signal-flow diagram of this model is shown in Figure 3. For a given triple \( (V_k, P_k, Q_k) \), the pair \( (x_k, \alpha_k) \) satisfying (3) is uniquely determined by \( x_k = [\delta_k, 0, E_k, V_{d,k}, 0, 0, 0]^T \) and \( \alpha_k \) in (9) with \( P_{m,k} = P_k \) where
\[ \delta_k = \angle V_i + \arctan \left( \frac{P_i}{Q_i + |V_i|^2} \right), \]
\[ V_{d,k} = \frac{X_{d,k}}{X_{d,k} + (X_{q,k} - 1)} \left| V_i \right| \cos (\delta_k - \angle V_i), \]
\[ E_k = \frac{|V_i|^4 + 2Q_i X_{d,k} + Q_i^2 X_{d,k}^2 + Q_i^2 X_{d,k} + P_i^2 X_{d,k}^2 + Q_i^2 X_{d,k}^2 + 2Q_i |V_i|^2 X_{d,k} + |V_i|^4}{|V_i|^2 X_{q,k}^2 + Q_i^2 X_{d,k}^2 + 2Q_i |V_i|^2 X_{d,k} + |V_i|^4}. \]

Remark 1

Relationships between the one-axis model described by (4) and (5) and other standard models of synchronous generators are shown in “Relationship Between the Standard Models of a Synchronous Machine.”

**Nonunit Buses**

Nonunit buses are simply modeled by the Kirchhoff’s power balance law, namely, for \( k \in \mathbb{N}_N \),
\[ P_k + j Q_k = 0. \]
For (1), \( x_k, u_k, \) and \( \alpha_k \) are empty vectors.

**Loads**

Loads are commonly modeled by algebraic power balance equations, although extensive literature also exists for dynamic loads [12], [13]. The well-known static load models are

- Constant impedance model: \( P + j Q = (z^{-1} V)^* V \)
- Constant current model: \( V = j z^{-1} P \)
- Constant impedance current model: \( V = j z^{-1} (P + j Q) \)
- Constant power factor model: \( V = \frac{j Q}{P} z^{-1} \)
- Constant impedance power factor model: \( V = \frac{j Q}{P} z^{-1} \)

**Brief Tutorial on Power Flow Calculation**

Given the number of buses \( N \) and an admittance matrix \( Y \), the power flow calculation, namely, a procedure of finding bus voltage \( V_k \), active power \( P_k \), and reactive power \( Q_k \) for \( k \in \{1, \ldots, N\} \) satisfies
\[ 0 = (YV_{i,N})^* - V_{i,N} - (P_{i,N} + jQ_{i,N}), \]
\[ V_{i,N} := [V_1 \cdots V_N]^T, \]
\[ P_{i,N} + jQ_{i,N} := [P_1 + jQ_1 \cdots P_N + jQ_N]^T. \]

The buses associated with \( \mathbb{N}_N, \mathbb{N}_L, \) and \( \mathbb{N}_E \) satisfying (S6) are called PQ buses because the steady-state values of their active and reactive power \( P \) and \( Q \), respectively, are given for the power flow calculation. For wind farms, solar farms, and generators, their steady-state active power and bus voltage magnitude are usually known. However, because the power loss through the transmission lines is not known a priori, active and reactive power of at least one component must be unspecified. Let this component be the generator connecting to the \( k \)-th bus for a given \( k \in \mathbb{N}_0 \), and assume
\[ 0 = [V_i] - V_{\delta_k}, \]
\[ 0 = \angle V_{\delta_k} - \delta_k. \]

For the loads and energy-storage systems, their steady-state active/reactive power \( P_k + j Q_k \) is known in advance. In view of this, \( h_v(\cdot, \cdot) \) for those units is
\[ 0 = [V_i] - V_{\delta_k}, \]
\[ 0 = \angle V_{\delta_k} - \delta_k. \]

For a given \( \mathbb{N}_N, \mathbb{N}_L, \) and \( \mathbb{N}_E \) satisfying (S8) are called PV buses because their steady-state values of the active power \( P \) and voltage magnitude \( V \) are given for the power flow calculation.

Finally, the power flow calculation is to find \( V_{i,N}, P_{i,N}, \) and \( Q_{i,N} \) satisfying (S3) and (S5)–(S8).
Excitation System Synchronous Machine

Exciter with AVR

Power System Stabilizer

Electromechanical Swing Dynamics

Electromechanical Dynamics

Figure 3 A signal-flow diagram of the model of a generator with its terminal bus, where the constant signals $P_m$, $V$, and $|V|$ are omitted.

Relationship Between the Standard Models of a Synchronous Machine

Four standard models of a synchronous machine are commonly used in power system modeling, depending on the desired resolution of the model [6]: the Park, subtransient, one-axis, and classical models. The mathematical relationship among these models is summarized as follows. The Park model is one of the best-known synchronous machine models, a combination of the motion dynamics and the electromagnetic dynamics representing the flux variation of the d- and q-axis circuits, excitation winding, and amortisseur windings. The equations of those two dynamics can be found in [6, Sec. 3.9] and [6, Sec. 3.4.9], respectively.

Assuming that the d- and q-axis circuit flux dynamics are sufficiently fast, the Park model can be simplified to the subtransient model with four coils on the rotor [7]. The model consists of the motion dynamics (a second-order system) and the fourth-order system representing the flux variation of the excitation windings, one d-axis amortisseur winding, and two q-axis amortisseur windings.

By further assuming that the amortisseur effects and the resistance between the generator and its connecting bus are negligible, the subtransient model can then be simplified to (4) and (5). This model is called the one-axis model [8], [S32] in the sense that its electromagnetic dynamics represent the flux variation of only the excitation winding.

In (S33), a further simplified one-axis model under the assumption that $X_{d,k} = X_{q,k}$ in (4) and (5) is presented. When $X_{d,k} = X_{q,k}$ (which can be satisfied for round rotor machines because of the symmetrical air gap between the d- and q-axes [10]), the electromagnetic dynamics of the simplified one-axis model is reduced to $T_{d,k} E_i = - E_i + V_{d,k}$. Thus, assuming the initial value of the internal voltage $E_i$ is its steady-state value $E_i$ and $V_{d,k}(t) = E_i$, it clearly follows that $E_i(t) = E_i$ for all $t$. In that case, the simplified one-axis model can be simply written as

$$
\frac{\delta_k}{\omega_k} = \frac{P_m}{M_k} - d_2 \omega_2 - \frac{V_i}{X_{d,k}} \sin(\delta_k - \gamma_k),
$$

$$
P_k + j Q_k = \frac{E_i}{X_{d,k}} \sin(\delta_k - \gamma_k) + \left[ \frac{E_i}{X_{q,k}} \cos(\delta_k - \gamma_k) - \frac{|V_i|}{X_{q,k}} \right].
$$

This model is called the classical model [6], [10]. The relationship among the four models is shown in Figure S2.

REFERENCES


constant current model \[ P + jQ = i^* V, \] (13)

constant power model \[ P + jQ = P + jQ, \] (14)

where \((i)^*\) is the complex conjugate operator, and \(z \in \mathbb{C}\), \(i \in \mathbb{C}\), and \(P + jQ \in \mathbb{C}\) are constant. For \(k \in \mathbb{N}_L\), a load at the \(k\)th bus can be represented by \(x_k\) and \(u_k\) being empty vectors; \(a_i\) being either \(z_k, i_k\) or \(P_i + jQ_i\); and the output equation being either (12), (13), or (14). Constant impedance loads will be used for the simulations in this article. For a given triple \((V_i, P_i, Q_i)\), the load impedance \(z_k\) will be uniquely calculated, such that \(P_i + jQ_i = (z_i^* V_i)^* V_i\).

**Wind Farms**

A wind farm model typically consists of a wind turbine, a doubly fed induction generator (DFIG), and a back-to-back (B2B) converter with associated controllers. A battery with a dc/dc converter can be added to the B2B converter, if needed. Figure 4 shows the physical architecture of a wind farm with its bus, while Figure 5 shows a signal-flow diagram of the model. When the battery is not connected, the current \(i_{dc}\) in Figure 4 is regarded as zero. The symbols for the wind farm model are listed in Table 5.

**WIND TURBINE**

The wind turbine, as shown in Figure 4, converts aerodynamic power from the wind to mechanical power that is transmitted to the DFIG. The turbine is typically modeled as a one-inertia or two-inertia model (the latter is followed in this article) consisting of a low-speed shaft, high-speed shaft, and gearbox [14]. For simplicity, the aerodynamic power \(P_i\) in the two-inertia dynamics (16) is assumed to be constant, as wind speeds usually change slowly.

**DOUBLY FED INDUCTION GENERATOR**

The DFIG in Figure 4 converts the mechanical power from the turbine into electrical power. The generator consists of a three-phase rotor and a three-phase stator. The stator is connected to the wind bus to transmit the electrical power into the grid while the rotor is connected to the B2B converter, with associated controllers that govern the rotor winding voltage. The stator and rotor are coupled electromagnetically, which is reflected in the dynamics of the stator and rotor currents expressed in a rotating d-q reference frame [15].

**BACK-TO-BACK CONVERTER WITH ITS CONTROLLERS**

The B2B converter is used for regulating the DFIG rotor voltages \(v_{dr}, v_{qr}\) and the reactive power flowing from the stator to the converter. The B2B converter consists of two three-phase voltage-source converters, namely, the rotor-side converter (RSC) and grid-side converter (GSC), linked via a common dc line [16]. Each of the converters is equipped with a controller. Here is an explanation of the models of the GSC, RSC, and their controllers.

» Following [16], the GSC dynamics are expressed as the variation of the ac-side current in the d-q reference frame.

» The GSC controller consists of inner-loop and outer-loop controllers [16]. The outer-loop controller generates a reference signal of the GSC currents \(i_{dc}, i_{qG}\) for regulating both the dc link voltage \(v_{dc}\) and the reactive power \(Q_r\) flowing into the GSC to their respective setpoints. The outer-loop controller is designed as a proportional-integral (PI) controller, as in (20). The inner-loop controller regulates \(i_{dc}, i_{qG}\) to the generated reference signals \(i_{dc,ref}, i_{qG,ref}\) by controlling the duty cycles \(m_{dc}, m_{qG}\). Following [16], the controller in this article is designed such that the transfer function from \(i_{dc,ref}\) (or \(i_{qG,ref}\)) to \(i_{dc}\) (or \(i_{qG}\)) is a desired first-order system \(1/(\tau_{CS} + 1)\) when the duty cycles are not saturated. The controller is implemented as (21).

» The RSC model is described as

\[
\begin{align*}
v_{dr} &= \frac{L_R}{d} i_{dr} + R_R i_{dr} - L_R i_{qr} + \frac{m_{dr}}{2} v_{dc}, \\
v_{qr} &= \frac{L_R}{d} i_{qr} + R_R i_{qr} + L_R i_{dr} + \frac{m_{qr}}{2} v_{dc},
\end{align*}
\] (15)

where \(i_{dr}\) and \(i_{qr}\) are the DFIG rotor currents, \(m_{dr}\) and \(m_{qr}\) are the duty cycles of the RSC, and \(v_{dc}\) is the dc link voltage. In this article, the RSC resistance and inductance are considered to be negligible, that is, \(R_R = L_R = 0\). This assumption is always satisfied by incorporating the two into the DFIG rotor circuit. Thus, the RSC model used in this article is described as (22).

» The RSC is equipped with inner-loop and outer-loop controllers. The outer-loop controller generates reference signals for the DFIG rotor currents \(i_{dr}\) and \(i_{qr}\) for regulating the stator voltage magnitude and...
high-speed shaft speed $\omega_r$ to their setpoints, while the inner-loop controller regulates the RSC currents [17]. This control action is actuated through the control of the duty cycles of the B2B converter.

The RSC and GSC are connected by a dc link equipped with a capacitor whose dynamics are derived from the power balance through the B2B converter [16].

**BATTERY AND DC/DC CONVERTER**

A battery is used for charging or discharging electricity as needed. The battery includes a dc/dc converter that steps the battery terminal voltage up or down. Both devices are sometimes used for suppressing the fluctuations in the output power $P + jQ$ by controlling the dc/dc converter. The model for each is described as follows.

**FIGURE 4** The physical structure of the model of a wind farm with its terminal bus. DFIG: doubly fed induction generator.

POD is one of the most critical real-time control problems in current power grids, and its importance is only going to increase with DER integration.
The overall control architecture for the future grid must be a combination of these decentralized plug-and-play DER controllers and distributed wide-area controllers.

The dc/dc converter is modeled by buck (step-down) and boost (step-up) models. These models are widely available in the literature [18]. When the converter dynamics are sufficiently fast, simpler models, where the output voltage and current are explicit functions of the duty ratio, can be derived. This simple model is used in this article.

The battery circuit is shown as the dark yellow part in Figure 4 [16]. Its dynamics can be represented as the variation of the battery voltage $v_b$ and output current $i_{dc}$.

**INTERCONNECTION TO GRID**

The net active and reactive power injected by the wind farm to the grid are determined as the sum of the power leaving the stator that is consumed by the B2B converter. The state-space representation of the overall wind farm model can be written as follows.

**Wind turbine**

$$\begin{align*}
J \dot{\omega}_r &= -(d_e + B_t) \omega_r + \frac{d_e}{N_e} \omega_r - K_e \theta_r + \frac{P_s}{\omega_r}, \\
J \dot{\omega}_r &= \frac{d_e}{N_e} \omega_r - \left(\frac{d_e}{N_e} + B_t\right) \omega_r + \frac{K_e}{N_e} \theta_r - T, \\
\dot{\theta}_r &= \dot{\omega}_m \left(\omega_r - \frac{1}{N_e} \omega_r\right),
\end{align*}$$

where $T$ is defined in (17).

**DFIG**

$$\begin{align*}
\dot{i} &= A(t) i + B [\text{Re}(V), \text{Im}(V)]^T + B_t [i_{dr}, i_{qr}]^T, \\
T &= X_m (i_{ds} i_{qr} - i_{qs} i_{dr}), \\
P_s + j Q_s &= \gamma_W (\text{Re}(V) i_{ds} + \text{Im}(V) i_{qr}) + j \gamma_W (\text{Im}(V) i_{ds} - \text{Re}(V) i_{qr}), \\
i &= [i_{dr}, i_{qr}, i_{ds}, i_{qs}]^T,
\end{align*}$$

**FIGURE 5** A signal-flow diagram of the model of a wind farm with its terminal bus, where constant signals $P_s, V_{dc}, Q_s, V, \omega_r$, and $\omega_i$ are omitted. DFIG: doubly fed induction generator; RSC: rotor-side converter; GSC: grid-side converter.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Numerical Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{ns}, \omega_{hs}$</td>
<td>Angular velocity of low-speed shaft and high-speed shaft</td>
<td></td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>Torsion angle (rad)</td>
<td></td>
</tr>
<tr>
<td>$P_a$</td>
<td>$2.45 \times 10^{-5}$</td>
<td>Aerodynamic power input, depending on wind speed</td>
</tr>
<tr>
<td>$J_s, J_r$</td>
<td>$1.95 \times 10^4, 0.138$</td>
<td>Inertia coefficients of the low-speed and high-speed shafts (seconds)</td>
</tr>
<tr>
<td>$B_s, B_r$</td>
<td>$9.87, 0.001$</td>
<td>Friction coefficients of the low-speed and high-speed shafts</td>
</tr>
<tr>
<td>$K_t$</td>
<td>508.9</td>
<td>Torsional stiffness (1/rad)</td>
</tr>
<tr>
<td>$d_c$</td>
<td>337.76</td>
<td>Damping coefficient of turbine</td>
</tr>
<tr>
<td>$N_g$</td>
<td>90</td>
<td>Gear ratio</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>$60\pi$</td>
<td>Mechanical synchronous frequency (rad/s)</td>
</tr>
<tr>
<td>$i_d, i_q$</td>
<td>d- and q-axis rotor currents</td>
<td></td>
</tr>
<tr>
<td>$i_{ds}, i_{qs}$</td>
<td>d- and q-axis stator currents</td>
<td></td>
</tr>
<tr>
<td>$v_{ds}, v_{qs}$</td>
<td>d- and q-axis rotor voltages</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>Electromechanical torque converted by DFIG</td>
<td></td>
</tr>
<tr>
<td>$P_s + jQ_s$</td>
<td>Power flowing from DFIG to bus</td>
<td></td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Number of wind generators inside the farm</td>
<td></td>
</tr>
<tr>
<td>$X_m$</td>
<td>197.64</td>
<td>Magnetizing reactance</td>
</tr>
<tr>
<td>$X_{ls}, X_{lr}$</td>
<td>4.620, 4.976</td>
<td>Stator and rotor leakage reactance</td>
</tr>
<tr>
<td>$R_s, R_r$</td>
<td>$0.244, 0.274$</td>
<td>Stator and rotor resistance</td>
</tr>
<tr>
<td>GSC and Its Controller</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_{ds}, i_{qs}$</td>
<td>d- and q-axis currents flowing from ac side to dc side</td>
<td></td>
</tr>
<tr>
<td>$m_{ds}, m_{qs}$</td>
<td>d- and q-axis duty cycles</td>
<td></td>
</tr>
<tr>
<td>$P_s + jQ_r$</td>
<td>Power flowing from bus to GSC</td>
<td></td>
</tr>
<tr>
<td>$Q_i$</td>
<td>0.001</td>
<td>Steady-state value of $Q_i$</td>
</tr>
<tr>
<td>$v_{dc}$</td>
<td>2.03</td>
<td>Steady-state dc link voltage</td>
</tr>
<tr>
<td>$\chi_{ds}, \chi_{qs}$</td>
<td>Inner-loop controller state</td>
<td></td>
</tr>
<tr>
<td>$\zeta_{ds}, \zeta_{qs}$</td>
<td>Outer controller state</td>
<td></td>
</tr>
<tr>
<td>GSC and Its Controller</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_{ds}, i_{qs}$</td>
<td>Reference signal of $i_{ds}$ and $i_{qs}$</td>
<td></td>
</tr>
<tr>
<td>$u_{du}, u_{qu}$</td>
<td>Additional control input signals</td>
<td></td>
</tr>
<tr>
<td>$L_{Gc}, R_{Gc}$</td>
<td>633.46, 0.05</td>
<td>Inductance and resistance of GSC</td>
</tr>
<tr>
<td>$K_{P,d}, K_{P,q}$</td>
<td>0.1, 0.01</td>
<td>P gains of outer controller</td>
</tr>
<tr>
<td>$K_{I,d}, K_{I,q}$</td>
<td>$1 \times 10^{-4}$, 0.001</td>
<td>I gains of outer controller</td>
</tr>
<tr>
<td>$\tau_{Gc}$</td>
<td>0.1</td>
<td>GSC current dynamics' time constant to be designed</td>
</tr>
<tr>
<td>RSC and Its Controller</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{dr}, m_{qr}$</td>
<td>d- and q-axis duty cycles</td>
<td></td>
</tr>
<tr>
<td>$\chi_{dr}, \chi_{qr}$</td>
<td>Inner-loop controller state</td>
<td></td>
</tr>
<tr>
<td>$i_{dr}^{<em>}, i_{qr}^{</em>}$</td>
<td>Reference signal of $i_{dr}$ and $i_{qr}$</td>
<td></td>
</tr>
<tr>
<td>$u_{dr}, u_{qr}$</td>
<td>Additional control input signals</td>
<td></td>
</tr>
<tr>
<td>$K_{P,dr}, K_{P,qr}$</td>
<td>0.01, 0.1</td>
<td>P gains of outer controller</td>
</tr>
<tr>
<td>$K_{I,dr}, K_{I,qr}$</td>
<td>5, 5</td>
<td>I gains of inner-loop controller</td>
</tr>
<tr>
<td>$K_{L,dr}, K_{L,qr}$</td>
<td>1, 1</td>
<td>I gains of inner-loop controller</td>
</tr>
<tr>
<td>DC Link</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{dc}$</td>
<td>dc-side voltage</td>
<td></td>
</tr>
<tr>
<td>$C_{dc}$</td>
<td>448.7</td>
<td>dc link capacitance</td>
</tr>
<tr>
<td>$G_{sw}$</td>
<td>$1.2 \times 10^{-5}$</td>
<td>Conductance representing switching loss of the B2B converter</td>
</tr>
<tr>
<td>Buck-and-Boost dc/dc Converter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_{dc}$</td>
<td>Current injecting from dc/dc converter into dc link</td>
<td></td>
</tr>
<tr>
<td>$v_{dc}$</td>
<td>Voltage at battery side</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>Step down/up gain</td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_b$</td>
<td>Battery voltage</td>
<td></td>
</tr>
<tr>
<td>$i_{dc}$</td>
<td>Current injected from the battery</td>
<td></td>
</tr>
<tr>
<td>$C_b$</td>
<td>$8.97 \times 10^3$</td>
<td>Battery capacity</td>
</tr>
<tr>
<td>$G_b$</td>
<td>$1.2 \times 10^{-5}$</td>
<td>Battery conductance</td>
</tr>
</tbody>
</table>
where $\omega_t$ is defined in (16), $v_{dr}$ and $v_{qr}$ in (22), and

$$
A_i(\omega_t) = \frac{1}{\beta} \begin{bmatrix}
-R, X_s & -\omega_t X_s X_t & R, X_m & -\omega_t X_s X_m \\
-\beta + \omega_t X_s X_t & -R, X_t & -\omega_t X_s X_t & R, X_s \\
-\omega_t X_s X_t & R, X_s & -\beta - \omega_t X_s X_t & -R, X_t \\
-\omega_t X_s X_t & R, X_s & -\omega_t X_s X_t & -R, X_t
\end{bmatrix},
$$

$$
B_i(\omega_t) = \frac{1}{\beta} \begin{bmatrix}
-X_s & 0 & 0 & X_m \\
0 & -X_s & 0 & X_m \\
X_m & 0 & -X_s & 0 \\
0 & X_m & 0 & -X_s
\end{bmatrix},
$$

$$
G_i = \frac{1}{\beta} \begin{bmatrix}
X_m & 0 & -X_s & 0 \\
0 & X_m & 0 & -X_s \\
X_s & X_m & 0 & -X_s \\
0 & X_s & X_m & 0
\end{bmatrix},
$$

$$
X_s := X_m + X_{tr}, \quad X_t := X_m + X_{dr}, \quad \beta := X_s X_t - X_m^2.
$$

where $m_{dc}$ and $m_{qdc}$ are defined in (21) and $v_{dc}$ in (25).

**GSC**

$$
\dot{\xi}_{dc} = K_{d,dc}(v_{dc} - v_{dc}^*),
$$

$$
\dot{\xi}_{qdc} = K_{q,dc}(v_{dc}^* - v_{dc}^*),
$$

$$
\dot{\xi}_{ac} = K_{a,dc}(v_{dc} - v_{dc}^*) + \xi_{ac},
$$

$$
\dot{\xi}_{qac} = K_{q,dc}(v_{dc}^* - v_{dc}^*) + \xi_{qac},
$$

where $Q_t$ and $V_{dc}$ are defined in (19) and (25).

**Inner-loop controller of the GSC**

$$
\tau_C \dot{\xi}_{dc} = \frac{m_{dc}}{\omega_t} - i_{dc},
$$

$$
m_{dc} = \text{sat}\left(\frac{\text{Re}(V)}{\omega_t} - \frac{m_{dc}}{\omega_t^2} i_{dc}^2 + \frac{m_{dc}}{\omega_t^2} i_{dc}\right).
$$

$$
\tau_C \dot{\xi}_{qdc} = \frac{m_{qdc}}{\omega_t} - i_{qdc},
$$

$$
m_{qdc} = \text{sat}\left(\frac{\text{Re}(V)}{\omega_t} - \frac{m_{qdc}}{\omega_t^2} i_{qdc}^2 + \frac{m_{qdc}}{\omega_t^2} i_{qdc}\right).
$$

where $i_{dc}$ and $i_{qdc}$ are defined in (19), $i_{dc}^*$ and $i_{qdc}^*$ in (20), and $v_{dc}$ in (25), and sat(·) is a saturation function whose output is restricted within the range of $[-1,1]$.

**RSC**

$$
v_{dr} = \frac{m_{dr}}{2} v_{dr},
$$

$$
v_{qr} = \frac{m_{qr}}{2} v_{qr},
$$

where $m_{dr}$ and $m_{qr}$ are defined in (24) and $v_{dc}$ in (25).

**Outer-loop controller of the RSC**

$$
i_{dr}^* = K_{r,dr}(\mid V \cdot \mid V'),
$$

$$
i_{qr}^* = K_{r,qr}(\omega_t - \omega_t),
$$

where $\omega_t$ is defined in (16).

**Inner-loop controller of the RSC**

$$
\dot{\chi}_{dr} = K_{dr}(i_{dr} - i_{dr}^*),
$$

$$
m_{dr} = \text{sat}\left(\frac{\text{Re}(V)}{\omega_t} (i_{dr} - i_{dr}^*) + \chi_{dr} + u_{dr}\right),
$$

$$
\dot{\chi}_{qr} = K_{qr}(i_{qr} - i_{qr}^*),
$$

$$
m_{qr} = \text{sat}\left(\frac{\text{Re}(V)}{\omega_t} (i_{qr} - i_{qr}^*) + \chi_{qr} + u_{qr}\right),
$$

where $i_{dr}$ and $i_{qr}$ are defined in (17), $i_{dr}^*$ and $i_{qr}^*$ in (23), and $v_{dc}$ in (25).

**DC link**

$$
\frac{C_{dc}}{\omega_t} \dot{v}_{dc} = \frac{1}{2v_{dc}} (\text{Re}(V) i_{ac} + \text{Im}(V) i_{dc}^* + v_{dr} i_{dr} + v_{qr} i_{qr}^* - R_{dc} (i_{dc}^* + i_{qdc}^*)) - G_{ov} v_{dc} + \frac{1}{2} i_{dc},
$$

where $i_{ac}$ and $i_{dc}$ are defined in (19), $v_{dc}$ and $v_{qr}$ in (22), $i_{dr}$ and $i_{qr}$ in (17), and $i_{dc}$ in (26). When the battery and dc/dc are not connected, $i_{dc} = 0$.

**Buck-boost dc/dc converter**

$$
v_{dc} = p(S + u_t) v_{dc},
$$

$$
i_{dc} = p(S + u_t) i_{dc},
$$

$$
p(x) = \begin{cases} x \text{ if } x \geq 0, \\ 0 \text{ otherwise,} \end{cases}
$$

where $v_{dc}$ and $i_{dc}$ are defined in (25) and (27).

**Battery**

$$
\frac{C_b}{\omega_t} \dot{v}_b = -i_{dc} - G_v v_b,
$$

$$
\frac{L_b}{\omega_t} i_{dc} = v_b - R_b i_{dc} - v_{dc},
$$

where $v_{dc}$ is defined in (26).

**Interconnection to grid**

$$
P + jQ = (P_r - P_i) + j(Q_r - Q_i),
$$

where $P_r$, $Q_r$, $P_i$, and $Q_i$ are defined in (17) and (28).

For (I), the wind farm model with the battery and dc/dc converter can be summarized as

$$
x_t := [\omega_t \omega_i \omega_r \theta_{1, \omega} \xi_{c, \omega} \xi_{q, \omega} \xi_{c, q} \xi_{q, c} \xi_{c, b} \xi_{q, b} v_{dc} v_{qr} i_{dc} i_{qdc}]^T \in \mathbb{R}^{18},
$$

$$
\xi_{c, b} := [i_{dc}, i_{qdc}, i_{dr}, i_{qdr}, i_{qr}, i_{dr}^*, i_{qdr}^*, i_{qr}^*] ^T,
$$

$$
\chi_{*} := [\chi_{dr}, \chi_{qdr}, \chi_{qr}, \chi_{qdr}^*],
$$

$$
\alpha_s := [v_{dr}, v_{qr}, V, \omega_t]^T \in \mathbb{R}^{5},
$$

and $f_s(\cdot, \cdot, \cdot)$ and $g_s(\cdot, \cdot, \cdot)$ in (I) follow from (16)–(28) for $k \in \mathbb{N}_W$. The steady-state $x_t$ is determined as follows. Note that, given the total generated power $P_i + Q_i$, there exists a degree of freedom for determining $P_{r, k} + jQ_{r, k}$ and $P_{i, k} + jQ_{i, k}$ satisfying (28) in a steady state. Thus, not only the triple $(V_i, P_i, Q_i)$ but also the pair $(P_{i, k}, Q_{i, k})$ must be known. In this setting, the pair $(x_t, \alpha_s)$ satisfying (3) is uniquely determined.

**Solar Farms**

A solar farm model consists of a PV array, buck-and-boost dc/dc converter, dc/ac converter with a controller, and dc link [19], as shown in Figure 6. The signal-flow diagram for the system is shown in Figure 7. The dynamics of the dc/ac converter, its controller, and dc link are similar to those in the wind farm model and are given in (32)–(35). The models of the PV array and dc/dc converter
are described as follows, and the symbols for the solar farm model are listed in Table 6.

**PHOTOVOLTAIC ARRAY**
The PV array is a parallel interconnection of $n_p$ circuits, each of which contains $n_s$ series-connected PV cells, as shown in Figure 8(a). Each PV cell is assumed to be identical. Typically, a PV cell has nonlinear $I$-$V$ characteristics, as shown by the blue line in Figure 8(b) [19]. Assuming that the PV cell is operated around the so-called maximum power point (MPP), where the cell output power is maximized, the $I$-$V$ curve around this point can be approximated by a linear function, as shown by the red line. In that case, the PV array can be modeled as a series connection of a constant voltage source with value $V_{PV} = n_vv_{eq}$ and a resistance whose value is $R_{PV} = (n_s/n_p)r_{eq}$. This PV array model is described as in (30).

**DC/DC CONVERTER**
A buck-and-boost dc/dc converter is used to ensure PV cell operation around the MPP. This can be done by determining the converter step down/up gain $S$ such that the steady-state PV array output voltage $v_{dc}$ and current $i_{dc}$ coincide with the maximum point on the $I$-$V$ curve. While

---

**FIGURE 6** An equivalent circuit of the model of a solar farm and its terminal bus. PV: photovoltaic.

**FIGURE 7** A signal-flow diagram of the model of a solar farm and its terminal bus, where the constant signals $P^*, Q^*$, and $S$ are omitted. PV: photovoltaic.
the gain can be dynamically regulated by controllers, for simplicity, the gain \( S \) is assumed to be constant. The dc/dc converter model is described as (31). The state-space representation of the overall solar farm model can be written as follows.

- **PV array**
  \[
  i_{d} = \frac{v_{PV} - v_{dc}'}{R_{PV}} \tag{30}
  \]
  where \( v_{dc}' \) is defined in (31).

- **Buck-and-boost dc/dc converter**
  \[
  \dot{v}_{dc} = Sv_{dc}, \quad i_{dc} = Si_{dc}' \tag{31}
  \]
  where \( v_{dc} \) and \( i_{dc}' \) are defined in (35) and (30).

- **DC/AC converter**
  \[
  \begin{align*}
  \frac{L_{dc}}{\omega} i_{d} &= -R_{dc} i_{d} + L_{dc} i_{q} + \text{Re}(V) - \frac{m_{d}}{2} v_{dc}, \\
  \frac{L_{dc}}{\omega} i_{q} &= -R_{dc} i_{q} - L_{dc} i_{d} + \text{Im}(V) - \frac{m_{q}}{2} v_{dc}, \\
  P + jQ &= -P_{PV} \left( \text{Re}(V) i_{d} + \text{Im}(V) i_{q} \right) - j P_{PV} \left( \text{Im}(V) i_{d} - \text{Re}(V) i_{q} \right) \tag{32}
  \end{align*}
  \]
  where \( m_{d} \) and \( m_{q} \) are defined in (34) and \( v_{dc} \) in (35).

- **Outer-loop controller of dc/ac converter**
  \[
  \begin{align*}
  \dot{\bar{x}}_{d} &= K_{i,d} (P' - P), \\
  \dot{i}_{d} &= K_{P,d} (P' - P) + \bar{\bar{\zeta}}_{d}, \\
  \dot{i}_{q} &= K_{P,q} (Q' - Q) + \bar{\bar{\zeta}}_{q} \tag{33}
  \end{align*}
  \]
  where \( P \) and \( Q \) are defined in (32).

- **Inner-loop controller of dc/ac converter**
  \[
  \begin{align*}
  \tau_{ac} \dot{v}_{dc} &= \hat{i}_{d} - i_{d}, \\
  m_{d} &= \text{sat} \left( \frac{2}{v_{dc}} \left( \text{Re}(V) i_{d} + \text{Re}(V) i_{q} \right) - \frac{L_{dc}}{\tau_{ac}} (i_{d}^* - \hat{i}_{d}) \right) + u_{d}, \\
  \tau_{ac} \dot{v}_{dc} &= \hat{i}_{q} - i_{q}, \\
  m_{q} &= \text{sat} \left( \frac{2}{v_{dc}} \left( \text{Im}(V) i_{d} + \text{Im}(V) i_{q} \right) - \frac{L_{dc}}{\tau_{ac}} (i_{q}^* - \hat{i}_{q}) \right) + u_{q} \tag{34}
  \end{align*}
  \]
  where \( i_{d} \) and \( i_{q} \) are defined in (32), \( i_{d}^* \) and \( i_{q}^* \) in (33), and \( v_{dc} \) in (35).

- **DC link**
  \[
  \frac{C_{dc}}{\omega} \dot{v}_{dc} = \frac{1}{2} \frac{C_{dc}}{v_{dc}} (\text{Re}(V) i_{d} + \text{Im}(V) i_{q} + v_{dc} i_{dc} - R_{dc} (i_{d}^* + i_{q}^*)) - G_{inv} v_{dc}, \tag{35}
  \]
  where \( i_{d} \) and \( i_{q} \) are defined in (32), and \( i_{dc} \) in (31).

For (1), the solar farm model can be summarized as

\[
\begin{align*}
\dot{x}_k := [i_{d,k}, i_{q,k}, \chi_{d,k}, \chi_{q,k}, \zeta_{d,k}, \zeta_{q,k}, v_{dc,k}] \in \mathbb{R}^7, \\
\dot{u}_k := [u_{d,k}, u_{q,k}] \in \mathbb{R}^2, \quad \alpha_k := [P_{i,k}, Q_{i,k}, S_{i,k}] \in \mathbb{R}^3, \tag{36}
\end{align*}
\]
and \( f_k(\cdot, \cdot, \cdot, \cdot) \) and \( g_k(\cdot, \cdot, \cdot) \) in (1) follow from (30)–(35) for \( k \in \mathbb{N}_S \). The steady-state value of \( x_k \) and \( S_k \) can be found as follows. Suppose that \((v_{dc,k}^*, i_{dc,k}^*) \) is at the MPP. Given

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Numerical Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_{d}, i_{q} )</td>
<td>d- and q-axis currents flowing from the ac side to the dc side</td>
<td>DC/AC Converter and Its Controller</td>
</tr>
<tr>
<td>( m_{d}, m_{q} )</td>
<td>d- and q-axis duty cycles</td>
<td></td>
</tr>
<tr>
<td>( P + jQ )</td>
<td>Power injecting from solar bus</td>
<td></td>
</tr>
<tr>
<td>( \chi_{d}, \chi_{q} )</td>
<td>Inner-loop controller state</td>
<td></td>
</tr>
<tr>
<td>( \zeta_{d}, \zeta_{q} )</td>
<td>Outer-loop controller state</td>
<td></td>
</tr>
<tr>
<td>( i_{d}^<em>, i_{q}^</em> )</td>
<td>Reference signal of ( i_{d} ) and ( i_{q} )</td>
<td></td>
</tr>
<tr>
<td>( u_{d}, u_{q} )</td>
<td>Additional control input signals on duty cycles</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{PV} )</td>
<td>Number of PV generators inside the farm</td>
<td></td>
</tr>
<tr>
<td>( L_{ac}, R_{ac} )</td>
<td>Inductance and resistance of dc/ac converter</td>
<td></td>
</tr>
<tr>
<td>( P' + jQ' )</td>
<td>Steady-state power injecting from solar bus</td>
<td></td>
</tr>
<tr>
<td>( K_{P,d}, K_{i,d} )</td>
<td>PI gains of d-axis outer-loop controller</td>
<td></td>
</tr>
<tr>
<td>( K_{P,q}, K_{i,q} )</td>
<td>PI gains of q-axis outer-loop controller</td>
<td></td>
</tr>
<tr>
<td>( \tau_{ac} )</td>
<td>Design parameter representing time constant of converter current dynamics</td>
<td></td>
</tr>
<tr>
<td>( v_{dc} )</td>
<td>dc link voltage</td>
<td>DC Link</td>
</tr>
<tr>
<td>( C_{dc} )</td>
<td>dc link capacitance</td>
<td></td>
</tr>
<tr>
<td>( G_{inv} )</td>
<td>Conductance representing switching loss of dc/ac converter</td>
<td></td>
</tr>
<tr>
<td>( i_{dc} )</td>
<td>Current flowing from dc/dc converter to dc link</td>
<td>Buck-and-Boost dc/dc Converter</td>
</tr>
<tr>
<td>( v_{dc} )</td>
<td>Voltage at PV array side</td>
<td></td>
</tr>
<tr>
<td>( S )</td>
<td>Step down/up gain so that solar farm is operated at MPP</td>
<td>PV Array</td>
</tr>
<tr>
<td>( R_{PV} )</td>
<td>Series resistance inside PV array model</td>
<td></td>
</tr>
<tr>
<td>( V_{PV} )</td>
<td>Voltage of constant voltage source inside PV array</td>
<td></td>
</tr>
</tbody>
</table>
The controllers should be modular by design and decentralized by implementation.

\[ V, P, Q, V', i_{dc,k} \text{ and } i_{dc,k} \text{ the pair } (x, \alpha) \text{ satisfying (3) is then uniquely determined as } x = [i_{d,k}, i_{q,k}, \chi_{d,k}, \chi_{q,k}, \zeta_{d,k}, \zeta_{q,k}, v'_{dc,k}]^T \text{ and } \alpha \text{ in (36), where} \]

\[
\begin{bmatrix}
\chi_{d,k} \\
\chi_{q,k} \\
\zeta_{d,k} \\
\zeta_{q,k}
\end{bmatrix}
= \begin{bmatrix}
\chi_{d,k} \\
\chi_{q,k} \\
\zeta_{d,k} \\
\zeta_{q,k}
\end{bmatrix}
= \frac{1}{|V|^2} \begin{bmatrix}
-\text{Re}(V_i) & -\text{Im}(V_i) \\
-\text{Im}(V_i) & \text{Re}(V_i)
\end{bmatrix}
\begin{bmatrix}
P_i \\
Q_i
\end{bmatrix},
\]

\[
v_{dc,k} = \sqrt{v_{dc,k}^2 + i_{dc,k}^2} - \frac{P_i}{Q_{PV,k} + R_{ac}(i_{d,k}^2 + i_{q,k}^2)}
\]

\[
S_k = \frac{v_{dc,k}^*}{v_{dc,k}}.
\]

**Energy-Storage Systems**

The energy-storage system consists of a battery, buck-and-boost dc/dc converter, dc/ac converter, and controller, as shown in Figure 9. The basic functions of these four components are to charge and discharge electricity, step down and step up the battery terminal voltage, rectify the three-phase current to a dc current, and regulate the dc voltage between the converters, respectively. When the energy-storage system is connected to the dc line, the dc/ac converter is not needed. The dynamics of the dc/ac converter, its controller, dc/dc converter, and dc link are similar to those described in (32), (33), (34), (31), and (35), respectively.

**FIGURE 8** (a) A photovoltaic (PV) array structure and (b) the I-V characteristics of PV cell KC200GT. MPP: maximum power point.

**FIGURE 9** The physical structure of the model of an energy-storage system with its terminal bus.
IMPACT OF DISTRIBUTED ENERGY RESOURCES ON
POWER SYSTEM DYNAMICS AND STABILITY

Given a DER-integrated power system model described by (1) and (2), the challenge is to determine how the penetration of DERs and their controllers dictates the stability and dynamic performance of the grid. This section demonstrates these impacts using numerical simulations of the IEEE 68-bus power system model [20]. The Matlab codes for these simulations can be found in [21]. The network diagram is shown in Figure 10. Each individual component is modeled by the equations listed in the previous section. The DER bus, denoted as bus 69, connects to bus 22. The reactance between buses 22 and 69 represents the transformer for stepping down the grid voltage to the DER voltage. Its value is taken as \( j0.01 \).

First, consider the DER to be a solar farm as in (30)–(35) with \( N_S = \{69\} \). The other bus indices \( N_G, N_L, \) and \( N_N \) are shown in Figure 10. The model of this PV-integrated power system is the combination of (1) and (2), where \( \Sigma_k \) for \( k \in N_G, N_L, N_N, \) and \( N_S \) is defined as (4)–(8), (12), (11), and (30)–(35). Note that \( \gamma_{PV} \) is the number of PV generators inside the farm. A question here is: How does \( \gamma_{PV} \) affect the small-signal stability of the grid? Small-signal stability is defined as the stability of the grid model linearized around its equilibrium [6].

The procedure to obtain the linearized model is as follows. First, compute an equilibrium of the entire system described by (1) and (2), as summarized in the previous section. Second, linearize the individual component dynamics of (1) and the interconnection of (2) around this equilibrium. Note that generators, solar farms, and wind farms are dynamic systems, while loads, nonunit buses, and the interconnection are static systems. Thus, the d- and q-axis voltages of all buses are redundant states. By eliminating these redundant states from the linearized differential algebraic equation model, a linearized ordinary differential equation model can be obtained. This method of elimination is referred to as Kron reduction.

FIGURE 10 An IEEE 68-bus, 16-machine power system model with one distributed energy resource (DER). The downward arrows represent load extractions.

Retrofit controllers can be added to or removed from the grid in a plug-and-play fashion without creating any sensitivity to other retrofit controllers.
Figure 11 shows the 13 dominant eigenvalues of this linearized power system model at a desired equilibrium. The eigenvalues around $-0.17 \pm j2$ for $\gamma_{PV} = 20$ start moving to the right as the value of $\gamma_{PV}$ increases and finally cross the imaginary axis when $\gamma_{PV} > 355$, resulting in an unstable system. Each PV generator is rated at 2 MW; therefore, $\gamma_{PV} = 355$ means that the net steady-state power output of the solar farm is $P_{PV} = 710$ MW, which is 3.85% of the total generated power of the system. This may appear to be a small percentage, but in terms of the stability limit the amount of solar penetration is quite close to critical. This pole shift occurs because the equilibrium changes with $\gamma_{PV}$. When a fault—modeled as an impulse function causing the initial conditions of $i_d, i_q$ in (32) to move from their equilibrium values—is induced, oscillations in the transient response of the states can easily be seen.

Figure 12 shows the frequency deviation of all 16 synchronous generators for the cases where $\gamma_{PV} = 20, 181, 306$. The results indicate that as $\gamma_{PV}$ increases, the PV-integrated power system, without any DER control, becomes oscillatory, with poor damping.

Next, the solar farm at bus 69 is replaced by a wind farm without a battery or dc/dc converter. Figure 13 shows the first 14 dominant eigenvalues of the linearized wind-integrated power system at a desired equilibrium. The eigenvalues around $-0.13 \pm 2.1j$ for $\gamma_{W} = 20$ start moving to the right as the value of $\gamma_{W}$ increases and finally cross the imaginary axis when $\gamma_{W} > 100$, resulting in an unstable system. Each wind generator is rated at 2 MW; therefore, $\gamma_{W} = 100$ means that the net steady-state power output of the farm is $P_{W} = 200$ MW. Thus, compared to the PV penetration, the wind penetration in this case poses a greater threat to small-signal stability.

To investigate the difference between the two, the singular value plot of the frequency response of each model from the d- and q-axis bus voltages $[\text{Re}(V), \text{Im}(V)]^T$ to the injected power $[P, Q]^T$ is shown in Figure 14(a) and (b),
respectively. The figure shows that the wind farm model has a resonance peak at 0.157 Hz, and the amplitude of the peak increases with $W_c$. This is an interesting observation, since 0.157 Hz lies in the range of frequencies for the low-frequency (0.1 to 2 Hz) oscillations of the synchronous generators, commonly called inter-area oscillations [22]. Thus, the wind injection at bus 69 stimulates an inter-area mode in this case. The resonance mode actually stems from the internal characteristics of the DFIG dynamic model. Details of this phenomenon can be found in [23]. The PV model, on the other hand, does not show any such resonance peak.

One potential way to combat the poorly damped oscillation would be to tune the PI gains of the converter controller described by (20) and (21) and by (23) and (24). However, such tuning must be done extremely carefully, with full knowledge of the entire grid model, since high values of these gains can jeopardize the closed-loop stability. This is shown in Figure 15, where the 10 dominant eigenvalues of the wind-integrated power system by changing the integral gains of the rotor-side converter controller of the back-to-back converter.

The variation of 10 dominant eigenvalues of the wind-integrated power system by changing the integral gains of the rotor-side converter controller of the back-to-back converter.

**FIGURE 14** A singular value plot of the frequency response from $[\text{Re}(V), \text{Im}(V)]^T$ to $[P, Q]^T$ for the (a) linearized solar farm model and (b) linearized wind farm model.

**FIGURE 15** The variation of 10 dominant eigenvalues of the wind-integrated power system by changing the integral gains of the rotor-side converter controller of the back-to-back converter.
follows. The dynamic system \( \dot{x} = f(x, u) \), where \( u \) is input, is said to be stable if the autonomous system under \( u = 0 \) is asymptotically stable. Consider a power system integrated with solar and wind farms. For \( k \in \mathbb{N}_S \cup \mathbb{N}_w \), let the dynamic model of the DER connected to the \( k \)th bus be rewritten as

\[
\dot{x}_k = A_k x_k + B_k u_k + \tilde{f}(x_k, V_k, u_k),
\]

where

\[
A_k := \frac{\partial f}{\partial x}, \quad B_k := \frac{\partial f}{\partial u},
\]

\[
\tilde{f}(x_k, V_k, u_k) := f(x_k, V_k, u_k; \alpha) - (A_k x_k + B_k u_k),
\]

and \( f(\cdot, \cdot, \cdot) \) follows from (30)–(35) for solar farms (when \( k \in \mathbb{N}_S \)) and from (16)–(28) for wind farms (when \( k \in \mathbb{N}_w \)). The following assumptions are imposed on the power system.

- Assumption 1: The power system model of (1) and (2) is stable.
- Assumption 2: The DER state vector \( x_k \) and its bus voltage \( V_k \) are measurable for each DER.

Assumption 1 is guaranteed by ensuring that the grid, without any additional controllers, is stable by properly tuning preexisting controllers, such as PSSs. Since PSSs guarantee the stable operation of all power systems in practice, this is a fair assumption for a retrofit controller to work in reality.

Assumption 2 simplifies the design; the availability of \( x_k \) can be relaxed to the output feedback case. The goal is to design a decentralized controller for each DER. The two main requirements from this controller are as follows:

1) The controllers should preserve closed-loop system stability and also improve the damping of the generator frequency deviations and line flows.
2) Each controller should depend on only local state feedback from its DER, not on any states from the rest of the grid (including other DERs). The controller should also be designed independent of the model of the rest of the system.

Property 2 implies that the controllers should be modular by design and decentralized by implementation.

The input \( u_k \) in (38) is composed of two parts, namely,

\[
u_k = u_{1,k} + u_{2,k}. \tag{39}\]

The component \( u_{1,k} \) assumes that \( \tilde{f}(\cdot, \cdot, \cdot) = 0 \) in (38). In other words, the design of \( u_{1,k} \) ignores the nonlinearity in (38), including the dependence on \( V_k \). This assumption is made only to simplify the design of \( u_k \). It does not influence the actual implementation of the control. Then, \( u_{1,k} \) in (39) can be simply designed as

\[
u_{1,k} = K_k x_k, \tag{40}\]

where \( K_k \) makes \( A_k + B_k K_k \) Hurwitz. However, in reality, this assumption will not be true. Thus, if \( u_k = u_{1,k} \) is implemented, then this control will pose serious threats to stability by neglecting the dynamics following from \( f(x_k, V_k, u_k) \) and neglecting the dynamics of the rest of the grid excluding the DER via \( V_k \), both of which will be stimulated by the control. To prevent this stimulation, a compensation signal \( u_{2,k} \) is designed for (39) using a dynamic compensator

\[
\Sigma_k : \begin{cases} \dot{x}_k = A_k \dot{x}_k + \tilde{f}(x_k, V_k, u_k) \\ u_{2,k} = -K_k \dot{x}_k \end{cases} \tag{41}\]

The final controller is written as the combination of (39)–(41) as

\[
\mathcal{R}_k : \begin{cases} \dot{x}_k = A_k \dot{x}_k + \tilde{f}(x_k, V_k, u_k) \\ u_k = K_k x_k - K_k \dot{x}_k \end{cases} \tag{42}\]

which is a retrofit controller. The following proposition holds for this controller.

**Proposition 1**

Let Assumptions 1 and 2 hold. The interconnection of the power system described by (1) and (2) and retrofit controllers \( \mathcal{R}_k \) in (42) for \( k \in \mathbb{N}_S \cup \mathbb{N}_w \) is stable for any \( K_k \) such that \( A_k + B_k K_k \) is Hurwitz.

The proof of Proposition 1 is shown in [4] and [23]. The signal-flow diagram of a DER equipped with the retrofit controller (42) is shown in Figure 16. Proposition 1 shows that the retrofit controllers satisfy the stability requirement in the earlier-listed property 1. Equation (42) shows that the controllers satisfy property 2, that is, \( \mathcal{R}_k \) can be designed using information from only \( f(\cdot, \cdot, \cdot) \) and \( x_k \) of the corresponding DER, which makes it modular. \( \mathcal{R}_k \) can be implemented using feedback from only \( x_k \) and \( V_k \), which makes it decentralized. Neither the model nor the states of the rest of the grid are needed for designing or implementing \( \mathcal{R}_k \). The controller gain \( K_k \) can be anything, as long as \( A_k + B_k K_k \) is Hurwitz. For example, given the DER model of (38), first design the state-feedback gain as \( K_k = -R_k^{-1} B_k^T X_k \), where \( X_k \) is a positive-definite matrix satisfying

\[
A_k^T X_k + X_k A_k - X_k B_k R_k^{-1} B_k^T X_k + W_k = 0,
\]

with suitable weight matrices \( R_k \) and \( W_k \), and thereafter design \( \mathcal{R}_k \) in (42). This linear-quadratic regulator (LQR)-based retrofit controller will be used in numerical simulations later. Note that the initialization of this controller can also be decentralized. The initial state \( \dot{x}_k(0) \) is the equilibrium of the DER to be controlled, that is, \( \dot{x}_k(0) = x_k \), where \( x_k \) can be computed in advance from the \( k \)th component dynamics under a solution of the power flow calculation. In other words, the initialization of the controller is completed without considering any of the other components.
Two important properties of the retrofit controller exist. In addition to the stability, the plug-and-play property of retrofit control can be used to improve the closed-loop dynamic performance of the grid by the proper selection of \( K_k \) in (42), which can help attenuate transient oscillations in the power flows. This will be shown in the numerical simulations. See [4] and [23] for theoretical details.

Also, retrofit controllers are most sensitive to faults that occur either at or near the DER bus because the closer a fault is to the DER bus, the more significant the change in the DER initial state from its equilibrium. If, on the other hand, a fault occurs far away from the DER bus, so as to cause practically no change in its initial state, then the retrofit controller will show no effect. This property implies that retrofit controllers can be added to or removed from the grid in a plug-and-play fashion without creating any sensitivity to other retrofit controllers. This modularity property of retrofitting will be illustrated by numerical simulations of the IEEE 68-bus system. See [23] for more theoretical details.

**Wide-Area Control**

Retrofit control is ideal for handling local disturbances. A separate layer of control is needed for handling disturbances that cause system-wide impacts on the entire grid. These controllers are called *wide-area controllers*. They are commonly actuated through additional control loops in the PSSs of synchronous generators. Ideally, retrofit control for designing wide-area controllers can also be considered. However, the assets in the legacy grid (excluding

**FIGURE 16** Signal-flow diagrams of distributed energy resources with the retrofit controller \( R_k \) in (42): (a) a wind farm and (b) a solar farm. The bus index \( k \) is omitted in the diagrams. PV: photovoltaic.
DERs) do not fluctuate much over longer terms and, therefore, do not necessarily need a plug-and-play-type modular control in excess of what is already provided by the conventional PSS. Thus, in practice, retrofitting may be overkill for WAC.

A typical WAC problem is formulated as follows. Consider a power system consisting of generators, loads, non-unit buses, and wind/solar farms. The entire system dynamics is described as (1) and (2). The variables \( p_k + jq_k \) and \( v_k \) for \( k \in [1, \ldots, N] \) are auxiliary variables that can be eliminated to convert the differential algebraic model into an ordinary differential equation model by Kron reduction [6]. By linearizing the Kron-reduced model about a desired equilibrium, the model of the power system is written in a compact form as

\[
\dot{x}_C = A_C x_C + B_C u_C + R_C \bar{x}_D, \quad \dot{x}_D = A_D x_D + B_D u_D + R_D \bar{x}_C, \quad (43)
\]

where \( \bar{x}_C \in \mathbb{R}^{n_G} \) and \( \bar{x}_D \in \mathbb{R}^{n_B + 18n_W} \) are the stacked representations of the generator state error and DER state error relative to the equilibrium \( x_i \) for \( k \in \mathbb{N}_G \) and \( k \in \mathbb{N}_S \cup \mathbb{N}_W \), respectively; and \( u_C \in \mathbb{R}^{n_G} \) and \( u_D \in \mathbb{R}^{n_B + 18n_W} \) are the stacked representations of \( u_k \) for \( k \in \mathbb{N}_G \) and \( k \in \mathbb{N}_S \cup \mathbb{N}_W \), respectively. The kth element of \( u_C \) represents a signal actuated through the PSS, as explained in (6). Ideally speaking, both \( u_C \) and \( u_D \) should be used for WAC. However, since the current penetration of DERs in most power systems is still quite small, most wide-area controller designs are based only on the generator models, ignoring the dynamics of the DERs. In other words, consider a model of the form

\[
\dot{\xi} = A_C \xi + B_C u_C, \quad \xi(0) = \bar{x}_C(0), \quad (44)
\]

A WAC problem for POD can then be defined as finding a gain matrix \( K_C \) such that

\[
u_C = K_C \xi, \quad K_C \in \mathcal{S} \quad (45)\]

minimizes

\[
J := \int_0^{\infty} (\xi_C^T(t) Q \xi_C(t) + u_C^T(t) R u_C(t)) \, dt \quad (46)
\]

for a given positive-semidefinite matrix \( Q \) and a positive-definite matrix \( R \), subject to (44). In (45), \( \mathcal{S} \subseteq \mathbb{R}^{n_G \times 18n_W} \) represents admissible controllers encapsulating the distributed nature of the controller.

Alternative formulations have also been proposed, an overview of which is summarized in “Brief Survey of Wide-Area Control.” Once \( K_C \) is designed, the WAC is implemented as \( u_C = K_C x_C \) in (43) by creating a sparse communication network between the designated set of generators. For simplicity, the communication is assumed to be ideal, that is, it does not have any delays or packet losses. Research on WAC under communication delays or packet losses can be found in [24] and [25]. The generator state vector \( x_C \) is assumed to be available (for state estimation of \( x_C \) using PMU measurements and decentralized Kalman filters [2], [26]–[29]). Owing to the wide availability of PMU data, utilities currently have reasonably good models for their operating regions. The independent system operators also have fairly accurate power system models. For a robust implementation of WAC, these models should be updated every few hours using fresh PMU data as the operating point changes, followed by an update in \( K_C \).

The goal of the constraint in (45) is to promote sparsity in \( K_C \) for minimizing the density of the underlying communication network without much sacrificing the closed-loop performance. The usual philosophy for constructing \( \mathcal{S} \) is as follows. For a natural number \( L \leq |N_G| \), consider a set of groups \( \mathcal{G} \subseteq \{1, \ldots, |N_G|\} \) such that \( \mathcal{G} = \{1, \ldots, |N_G|\} \). Note that the groups are not necessarily disjoint, namely, there may exist a pair \((l, j)\) such that \( l \in \mathcal{G}_l \cap \mathcal{G}_j \neq \emptyset \). Let \( K_{C,l} \in \mathbb{R}^{1 \times L} \) denote the \((l, j)\) block matrix of \( K_C \). Define \( \mathcal{S} \) as

\[
\mathcal{S} := \{K_C : K_{C,l} = 0, \text{ there does not exist } l \in [1, \ldots, L], \text{ such that } i \in \mathcal{G}_l \land j \in \mathcal{G}_j\}. \quad (47)
\]

The problem, then, is to find \( K_C \) in (45) minimizing (46) under the constraint \( \mathcal{S} \), as in (47). The (sub)optimal set of groups \( \{\mathcal{G}_l\}_{l=1}^L \) and the structured feedback gain \( K_C \) can be constructed in different ways depending on the objective of the controller. For POD problems, operators are often interested in damping only the inter-area oscillation modes. In that case, \( \mathcal{G}_l \) can be chosen in the following way, as recently shown in [5]. Modeling the fault as an impulse input, let the small-signal impulse response of any generator frequency deviation be written as

\[
\Delta \bar{\omega}_i(t) = \sum_{j=1}^K (\alpha_i \exp(\lambda_j t) + \alpha_i^* \exp(\lambda_j^* t))
\]

\[
\text{inter-area modes}
\]

\[
+ \sum_{l=L+1}^{n_l} \beta_i \exp(p_i t) + \beta_i^* \exp(p_i^* t)), \quad (48)
\]

\[
\text{other modes}
\]

for a given positive-semidefinite matrix \( Q \) and a positive-definite matrix \( R \), subject to (44). In (45), \( \mathcal{S} \subseteq \mathbb{R}^{n_G \times 18n_W} \) represents admissible controllers encapsulating the distributed nature of the controller.

Future power grids must have both control architectures implemented on top of each other to enjoy their combined benefits.
The modularity of retrofit controllers will enable smoother integration of renewables into the grid in a plug-and-play fashion without jeopardizing stability or having to retune the existing PSS gains.

where \( k \) is the number of local modes (explained later), \( n \) is the dimension of the entire system, and \( \alpha_i, \lambda_i, \beta_i, \rho_i \) are modal coefficients. Assuming that the other modes are sufficiently damped by PSSs (as a result of which their effect dies down quickly), the goal is to add damping to only the inter-area oscillation modes.

The dominance of the inter-area modes is defined based on the magnitude of the modal coefficients \( \alpha_i \). For example, consider a power system that includes four generators (namely, \( |N_G| = 4 \)), with three inter-area modes (namely, \( k = 3 \)). Let the residues \( \alpha_{11}, \alpha_{21}, \alpha_{31}, \alpha_{32}, \alpha_{33} \) be classified as dominant residues because they satisfy \( |\alpha_i| \geq \mu \), where \( \mu \) is a prespecified threshold. In other words, it is assumed that the inter-area modes \( \lambda_1, \lambda_2 \) are substantially excited by the incoming disturbance, while the third inter-area mode has much poorer participation in the states. From the indices of the dominant modes, construct the two sets

\[
G_1 = \{1, 2, 3\}, \quad G_2 = \{3, 4\},
\]

which indicate that the generators in the first group participate dominantly in \( \lambda_1 \) and those in the second group participate dominantly in \( \lambda_2 \). This grouping information is then used to decide the topology of communication. In general, the generators inside the \( i \)th group should communicate with each other for suppressing the amplitude of oscillations excited by the mode \( \lambda_i \). The mode \( \lambda_3 \) for the above example is poorly excited, and, therefore, is ignored in the control design. The structure of \( K_c \) for this four-machine example is then constructed as (45), with the structure constraint (47) defined by the group set (49). For this example, the controllers can be written as

\[
\mathcal{K}_1: \quad u_1 = K_{c,11} \dot{x}_1 + K_{c,12} \dot{x}_2 + K_{c,13} \dot{x}_3,
\]

\[
\mathcal{K}_2: \quad u_2 = K_{c,21} \dot{x}_1 + K_{c,22} \dot{x}_2 + K_{c,23} \dot{x}_3,
\]

\[
\mathcal{K}_3: \quad u_3 = K_{c,31} \dot{x}_1 + K_{c,32} \dot{x}_2 + K_{c,33} \dot{x}_3 + K_{c,34} \dot{x}_4,
\]

\[
\mathcal{K}_4: \quad u_4 = K_{c,41} \dot{x}_3 + K_{c,44} \dot{x}_4.
\]

Because of the sparse structure of the communication network, the controllers are implemented in a distributed manner, as opposed to all-to-all communication that would be equivalent to a centralized implementation. Finally, the nonzero entries of \( K_c \) are computed using suboptimal structured LQR algorithms, such as \( L_1 \)-sparse optimal control via ADMM [30].

**Combined Control Architecture for Tomorrow’s Grids**

The relative advantages of retrofit control versus WAC lies in their effectiveness regarding different types of faults and fault locations. Retrofit control of DERs, for example, is more effective than WAC when a fault that changes the DER initial state from its equilibrium occurs either at or closer to the DER bus. This is because the former has a high controllability of the DER states, while the latter (being actuated through the AVR and PSS of synchronous generators) has a much lower controllability of those states. Similarly, WAC is much more effective than retrofit control when a fault occurs at non-DER buses.

Thus, to accommodate all types of fault scenarios, the control architecture for future grids must be a combination of two layers: a completely decentralized retrofit control mechanism for each individual DER and a distributed, peer-to-peer communication-based WAC between the synchronous generators. The modularity of retrofit controllers will enable smoother integration of renewables into the grid in a plug-and-play fashion without jeopardizing stability or having to retune the existing PSS gains. WAC, on the other hand, will be necessary to balance the increasing dynamic interdependence of grid components that may be geographically distant but are electrically close due to the installation of new transmission lines. The combination of these two control layers is shown in the form of an architectural diagram in Figure 17.

**Numerical Simulations**

This section demonstrates the effectiveness of the combined retrofit and WAC through simulation of the IEEE 68-bus power system model with DERs. The Matlab codes for all simulations have been made public in the repository [21]. First, we investigate the effectiveness of the retrofit control. Let a single wind farm be connected to bus 69, as shown in Figure 10. Let \( \nu_w = 60 \). The behavior of this wind-integrated power system after a fault (modeled as an impulsive change in the angular velocity of the low-speed shaft of the wind turbine) is simulated. Let \( \omega(0) = 0.3\omega_i \).

In Figure 18(a)–(e), the red lines show the trajectories of the DFIG rotor speed \( \omega_r \) in (16), the DFIG stator currents \( i_d \) and \( i_q \) in (17), the active and reactive power \( P \) and \( Q \) in (28) injected by the wind farm, the GSC duty cycles \( m_{AC} \) and \( m_{QC} \) in (21), and the frequency deviation \( \Delta\omega \) in (4) of all of the synchronous generators. The fault causes a decrement in the rotor speed \( \omega_r \) and induces oscillations in the DFIG
stator currents due to the resonance phenomenon described earlier. Both the active power $P$ and reactive power $Q$ injected into the grid from the wind farm start oscillating, which, in turn, induces oscillations in the generator frequencies. Thus, although the grid is small-signal stable, the impact caused by the fault makes the system leave its domain of attraction, resulting in transient instability.

The retrofit controller (42) is next used to counteract this instability. Since the instability is caused by the oscillations of the DFIG currents, the state feedback gain $K_k$ is designed to attenuate these oscillations using an LQR controller. In Figure 18(a)–(e), the blue lines show the trajectories of $\omega_r$, $i_{ds}$, $i_{qs}$, $P$ and $Q$, $m_{dc}$ and $m_{qc}$, and $\Delta \omega$ of the closed-loop system, respectively. By comparing the red lines and blue lines in Figure 18(b) and (c), it can be seen that the oscillations in the stator current and output power are significantly mitigated by retrofit control. As a result [as shown in Figure 18(e)], the oscillations in the frequency deviations are also reduced. However, the oscillations reappear after $t > 150$ s as the duty cycles $m_{dc}$ and $m_{qc}$ of the GSC in (21) become saturated, which is shown in Figure 18(d). Since the duty cycles are inversely proportional to $v_{dc}$, this saturation tends to occur when the dc link voltage is small. Furthermore, as shown in Figure 18(a) and (c), there exist negative offsets in $\omega_r$ and $P$. This implies that some of the mechanical power and output wind power are less than their desired values because of the decrement of the low-speed shaft speed.

Both of these shortcomings can be resolved by adding a battery to the dc link, which compensates for the dc-link voltage variation and discharges energy to make up for the insufficient power. Figure 19(f) shows that the battery voltage declines as the battery discharges stored energy. Comparing the blue lines in Figure 18(a) and (c) with those in Figure 19(a) and (c), it can be seen that the rotor speed and injected wind power now both converge to their respective setpoints. Furthermore, the duty cycles are no longer saturated, as the dc-link voltage remains almost constant. The battery voltage converges to its setpoints value asymptotically.

Over time, new DERs will be added to the existing grid. To emulate this, a solar farm with its bus (denoted as bus 70)

![Figure 17](image-url)

**Figure 17** The grid control architecture showing the coexistence of local and wide-area controllers for a four-machine power system with four distributed energy resources (DERs). The local controller $R_k$ is designed in (42). The wide-area controller $K_G$ in (45) is designed for the two groups $\mathcal{G}_1 = \{1, 2, 3\}$ and $\mathcal{G}_2 = \{3, 4\}$. The sparse structure of wide-area control avoids the need for all-to-all communication between the generators.
is added at bus 66, as shown in Figure 20. The system behavior is investigated after installing the second retrofit controller at this solar farm while retaining the first one at the wind farm. The design of the second retrofit controller follows the same procedure as the first. Figure 21 shows the trajectories of all of the generator frequency deviations when a fault occurs at the solar farm (fault 1). Figure 21(b) shows the control input calculated by the first and second retrofit controller. It can be seen that the first retrofit controller is inactive in this situation, meaning that it does not have any influence on the closed-loop response. The second retrofit controller, on the other hand, improves damping as soon as it is activated, as depicted by the blue solid lines in Figure 21(a). This shows that retrofit controllers can be

![Figure 18](image-url)

**Figure 18** (a) Trajectories of the doubly fed induction generator (DFIG) rotor speed, (b) the DFIG stator currents, (c) the active and reactive power injected from the wind farm, (d) the grid-side converter duty cycles, and (e) the frequency deviation of all synchronous generators.
added to or removed from the grid in a plug-and-play fashion without creating any sensitivity to other retrofit controllers. The design enjoys a natural decoupling property from one DER to another.

When the fault occurs at the DER bus, the retrofit controller alone is sufficient for mitigating oscillations. Any additional WAC action may not be necessary. To demonstrate this, an LQR-based wide-area controller is designed for all 16 synchronous generators and actuated through their PSSs following the fault at the wind farm (fault 2). Figure 22(a) summarizes the different ways in which the power system reacts to this fault. Since the fault occurred at the DER bus, the retrofit controller at the DER in case 2 successfully cancels its adverse effect and mitigates the oscillations in both

**FIGURE 19** (a) Trajectories of the doubly fed induction generator (DFIG) rotor speed, (b) the DFIG stator currents, (c) the active and reactive power injected from the wind farm, (d) the grid-side converter duty cycles, (e) the frequency deviation of all synchronous generators, and (f) battery voltage.
the generators and the DER, as shown in (a21) and (a22) in Figure 22.

If only WAC is used in this situation without any retrofit at the DER (which is case 3 in the figure), then an interesting observation is made: the generators are damped well, but the DER response is still unacceptably oscillatory. This clearly shows that WAC has limited controllability of the DER states, and, thus, for this situation, using only WAC is not going to

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**FIGURE 20** An IEEE 68-bus, 16-machine power system model with two distributed energy resources. A single wind farm (with bus 69) is added to bus 22, while a single solar farm (with bus 70) is added to bus 66.

**FIGURE 21** (a) The trajectories of the frequency deviation of all synchronous generators. The first and second retrofit controllers are implemented on the wind and solar farms at buses 69 and 70, respectively. (b) The trajectory of the control input \( u_k \) generated by the first and second retrofit controllers when fault 1 occurs.
suppress all oscillations. Retrofit control is absolutely imperative for this case. The respective responses are shown in (a31) and (a32) in Figure 22. However, when the fault happens outside the wind farm, then the retrofit controller is completely inactive, and the wide-area controller becomes necessary. To show this, a three-phase fault is induced at bus 10 (fault 3), with a fault clearing time of 0.07 s. By comparing Figure 22(b11)–(b32), it can be seen that the retrofit controllers at the wind and solar farms are no longer effective, whereas the wide-area controller successfully damps the power flow oscillations.

Finally, in Figure 22, (a41) and (a42) and (b41) and (b42) show the case when the two retrofit controllers and a wide-area controller are used together in the system. The combination can now handle faults occurring at both DER buses and non-DER buses. The simulations bear a clear message that future power grids must have both control architectures implemented on top of each other to enjoy their combined benefits.

**CONCLUSIONS AND FUTURE WORKS**

With the proliferation of distributed renewable generation, many interesting opportunities for control and optimization arise in power system research. On the one hand, WAC is becoming essential to making the grid more resilient against blackouts. Alternatively, local plug-and-play-type controllers are becoming essential for renewable energy sources. This article proposed a control architecture that combines these two types of controllers, highlighting various design and implementation challenges and solutions for both. The vision is that this architecture will serve as a platform for control theorists and power engineers to work together to create a sustainable and secure future for electric energy supply in every corner of the world. A list of open questions is presented for future work.

Not only the level of penetration of DERs but also their relative locations in the grid may have a significant influence on power system stability. Referencing the simulation example presented in the article, when a wind farm with an output

**FIGURE 22** The trajectories of the frequency deviation of all synchronous generators and power injecting from the wind farm when (a) fault 2 happens at the wind farm and (b) fault 3 (a three-phase fault at bus 10) occurs and when the following are used: (a11, a12, b11, and b12) no additional controllers, (a21, a22, b21, and b22) two retrofit controllers, (a31, a32, b31, and b32) a wide-area controller, and (a41, a42, b41, and b42) both two retrofit controllers and a wide-area controller.
power of 200 MW is connected to bus 22 of the IEEE 68-bus model, the power system model becomes unstable. On the other hand, if a wind farm with an output power of only 58 MW is connected to bus 42, the system becomes unstable. More work is needed to understand the system-level characteristics of power system models that decide how the spatial distribution of DERs may or may not preserve stability.

The controllers outlined in this article address transient stability and damping performance. In an actual grid, however, many other parallel control mechanisms will also exist—for example, load frequency control (LFC), which maintains the grid frequency at the synchronous value despite fluctuations in loads. LFC is designed independent of WAC and DER control because of its slower time scale. However, with gradual disintegration of the grid into smaller microgrids, this time-scale separation may become less dominant, leading to a stronger coupling between the controllers. The positive and negative effects of this coupling need more research.

Many open challenges exist for WAC as well. For example, instead of updating $A_G$ and $B_G$ in (43) with new PMU data every few hours and redesigning $K_G$ in (45), a future option can be to learn $K_G$ directly after a contingency using online reinforcement learning, Q-learning, adaptive dynamic programming, and similar model-free learning methods. This online learning approach will become important if the contingency significantly changes the nominal $A_G$ and $B_G$. Furthermore, both $u_G$ and $u_D$ in (43) must be used for WAC design in the future grid to accommodate the coupling effects between $x_G$ and $x_D$. Recent work, such as [31], reported similar preliminary results, where wind power controllers were used in conjunction with conventional PSS controllers for wide-area damping control action. More work is required, especially with regard to the time-scale separation between $u_G$ and $u_D$ and the sensitivity between the retrofit and WAC parts of $u_D$.

All of these questions deserve dedicated attention from researchers with backgrounds in control theory, power systems, signal processing, computer science, economics, and machine learning.

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